JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 35, No. 4, July–August 1998

Calculation of Sensitivities in Thermal Control Systems with Nonlinear Inequality Constraints

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The design of a thermal system is studied using temperature sensitivities. The thermal system is subjected to nonlinear temperature constraints, which are in the form of inequalities. Temperature sensitivities and the derivatives of the temperature constraints with respect to different design parameters are calculated by different methods, and their relative computational efforts are discussed. The thermal system pertaining to a shunt regulator in a spacecraft is considered. For thermal sensitivity analysis, design parameters such as internal heat dissipation, optical properties of an optical solar reflector, emittance of black paint, and emittance of low-emittance tape are considered. The numerical results indicate that the absorptance of the optical solar reflector is the most important parameter in the thermal control of a shunt regulator. An appropriate thermal shield and an increase in the black painted area are suggested as suitable design strategies to control the temperature of the shunt regulator.

Nomenclature

A	= total surface area of the nodal element exposed to solar radiation, m^2
A_c	= cross-sectional area of the nodal element, m^2
A_s	= surface area of the nodal element, m^2
$\boldsymbol{B}$	= matrix representing the temperature sensitivity functions as defined in Eq. (19), K/unit of the parameter considered
$\boldsymbol{C}$	= matrix having components $\partial f_i/\partial \beta_j$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, M$, as defined in Eq. (19), W/unit of the parameter considered
$F_{i,j}$	= geometrical view factor between the ith and jth nodes
$\boldsymbol{f}$	= function defined in Eq. (5), W
G_i	= temperature constraint function for the ith node as defined in Eq. (6), K
g_i	= heat generated at the ith node, W
K_{x_i}	= thermal conductivity of the material in the x_i direction, $i = 1, 2, 3$, W/m K
$l_{i,j}$	= linear elemental length between ith and jth nodes, m
M	= total number of design parameters considered
N	= total number of nodes in the system
$\boldsymbol{P}$	= row matrix as defined in Eq. (21)
s	= solar intensity, W/m^2
$\boldsymbol{T}_i$	= temperature of the ith element or node, K
(x_1, x_2, x_3)	= Cartesian coordinates of a point in space, m
α	= solar absorptance of the surface
$\boldsymbol{\beta}_j$	= jth design parameter
$\boldsymbol{\beta}_{j0}$	= nominal value of the design parameter $\boldsymbol{\beta}_j$
$\Delta \boldsymbol{T}_i$	= variation of the temperature of the ith node, K
$\Delta \boldsymbol{\beta}_j$	= arbitrary small variation in the design parameter $\boldsymbol{\beta}_j$
ε	= infrared emittance of the surface
θ	= angle between surface normal and the sun vector
σ	= Stefan–Boltzmann constant, W/m^2 K^4
$\boldsymbol{\varphi}$	= vector as defined in Eq. (23)

Subscripts

i, j, k	= ith, jth, and kth nodes
l, m	= lth and mth elements
sp	= space

Superscripts

T	= transpose of a matrix
-1	= inverse of a matrix

Introduction

CONTROL of high heat dissipation within electronic packages and the external heat loads on them from the surroundings is one of the major concerns of a thermal designer inasmuch as the temperatures of the components of the electronic packages are to be maintained in a specified range. This specified temperature range, when transformed into a mathematical model, results in nonlinear inequality constraints for temperatures. The present study pertains to a shunt regulator in a spacecraft system. The objective is to suggest an optimal design for the shunt regulator by taking into consideration the variations in the design parameter values, which in turn change the temperatures.

Thermal sensitivity studies of spacecraft systems, as well as some other structures, have been the subject of many investigations. They are essentially of two types: In the first, the temperature sensitivities have been calculated without temperatures being subjected to constraints, whereas in the second, thermal sensitivity studies have been conducted incorporating the constraints imposed on the system. Goble[1] discussed the steady-state temperature sensitivities associated with a Skylab orbital assembly without considering the constraints on temperatures. Similar studies have been conducted by Suresha et al.[2] in connection with a transient problem concerning a spacecraft battery.

Haftka[3] described the numerical techniques for computing the sensitivities of temperatures when the system is subjected to linear constraints. Haftka[4] suggested a method to improve the accuracy of the calculated sensitivities. Haftka and Malkus[5] considered large thermal systems with thousands of degrees of freedom and suggested ways of computing thermal sensitivities. They also discussed methods to reduce computational cost and errors in sensitivity calculations. Belegundu[6] discussed design sensitivity analysis of thermal systems with constraints. The adjoint variable method and Lagrangian approach were considered, and it was pointed out that the final equations obtained from the Lagrangian approach are

Received Aug. 19, 1997; revision received March 10, 1998; accepted for publication March 30, 1998.

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identical to those obtained from the adjoint method. House et al.[7] conducted the design sensitivity analysis of optimal control problems with constraints and compared the direct differentiation and adjoint methods.

In the present study, a steady-state system in spacecraft environment, namely, a shunt regulator of the Indian National Satellite System, is considered. The system is subjected to nonlinear inequality temperature constraints. Temperature sensitivities of some of the components of the shunt regulator have been analyzed with respect to different design parameters, such as internal heat dissipation, optical properties of an optical solar reflector (OSR), emittance of black paint, and emittance of low-emittance tape. Different methods have been used for the calculation of the derivatives of the constraints with respect to different design parameters. The relative efficiency of these methods has been discussed. Instead of considering the heat balance equation in its original form throughout the volume of each element, a lumped parameter approach has been used to construct a computational model in which the entire system is divided into a number of isothermal nodes. This model results in a system of coupled nonlinear algebraic equations in which the temperatures at the nodes are unknowns. This nonlinear algebraic system is to be solved subject to satisfying nonlinear inequality temperature constraints. It was found that, as compared to other parameters, the effect of the absorptance of the OSR is more important in controlling the temperatures of the components. Based on the computation of temperature sensitivities, design modifications have been suggested for the shunt regulator in the form of a thermal shield and an increase in the black-painted radiating area.

Problem Formulation

In thermal sensitivity analysis, the lumped parameter approach is generally considered for numerical computations. It is extremely difficult to deal with an energy balance in the differential form within the components, which may have different geometries with complicated boundary conditions on their surfaces. For the sake of completeness, some of the steps in the lumped parameter approach are given here, and further details can be obtained from a recent paper.[2]

A system in a spacecraft environment may consist of several solid elements, which are subjected to conduction and radiation modes of heat transfer. Conduction heat transfer occurs within each solid element, and radiative heat transfer occurs at the surfaces of the solid elements. In addition, internal heat generation, heat dissipation to space, solar radiation, etc., are present. A steady-state system has been considered in the present study. Using Fourier's heat conduction law, the energy balance for the mth solid element can be written as

$$K_{x_1}\frac{\partial^2 \boldsymbol{T}_m}{\partial x_1^2} + K_{x_2}\frac{\partial^2 \boldsymbol{T}_m}{\partial x_2^2} + K_{x_3}\frac{\partial^2 \boldsymbol{T}_m}{\partial x_3^2} + g_m(x_1, x_2, x_3) = 0 \quad (1)$$

In Eq. (1), heat conduction is spatially orthotropic, and $g_m(x_1, x_2, x_3)$ is the source term. The boundary conditions on the surfaces of this element can be taken in the form

$$-K_{x_i}\frac{\partial \boldsymbol{T}_m}{\partial x_i} = \sum_{l=1}^{N} \sigma\varepsilon_m F_{m,l} A_{s_m}\left(\boldsymbol{T}_l^4 - \boldsymbol{T}_m^4\right)$$
$$- \sigma\varepsilon_m F_{m,\mathrm{sp}} A_{s_m} T_m^4 + \alpha_m s A_m \cos\theta$$
$$m = 1, 2, \ldots, N, \qquad i = 1, 2, 3 \quad (2)$$

The first term on the right-hand side of Eq. (2) represents the radiative heat transfer to other elements of the system. The second term represents the radiative heat transfer to outer space in which the space temperature is taken to be 0 K. The third term defines the incident solar heat energy on the surfaces of the solid element. The albedo and Earth shine loads have been neglected because their contribution is considered insignificant as compared with that of direct solar load. For computational purposes, each solid element is subdivided into numbers of subelements called nodal elements. Each nodal element has a definite volume, and the thermal energy of the entire nodal element is assumed to be concentrated at a node that is attached to the nodal element. This may lie inside or on the surface of the nodal element, depending on the nature of heat transfer taking place between a specific subelement and its neighboring elements. If an energy balance equation is written at each node, the following coupled nonlinear algebraic equations can be obtained from Eqs. (1) and (2):

$$\sum_{j=1}^{N}\frac{K_{i,j}}{l_{i,j}} A_{c_j}(\boldsymbol{T}_j - \boldsymbol{T}_i) + \sum_{j=1}^{N}\sigma\varepsilon_i F_{i,j} A_{s_i}\left(\boldsymbol{T}_j^4 - \boldsymbol{T}_i^4\right)$$
$$+ g_i + \alpha_i s A_i \cos\theta = 0, \qquad i = 1, 2, \ldots, N \quad (3)$$

The preceding equations can be written in a general form as

$$f_i = f_i(\boldsymbol{T}_1, \boldsymbol{T}_2, \ldots, \boldsymbol{T}_N, \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_M) = 0$$
$$i = 1, 2, \ldots, N \quad (4)$$

or more compactly as

$$f_i(\boldsymbol{T}, \boldsymbol{\beta}) = 0, \qquad i = 1, 2, \ldots, N \quad (5)$$

where $\boldsymbol{T}$ and $\boldsymbol{\beta}$ are vectors with components

$$\boldsymbol{T} = [\boldsymbol{T}_1, \boldsymbol{T}_2, \ldots, \boldsymbol{T}_N], \qquad \boldsymbol{\beta} = [\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_M]$$

The temperatures $\boldsymbol{T}_1, \boldsymbol{T}_2, \ldots, \boldsymbol{T}_N$ of the nodes may also have to satisfy some inequality temperature constraints. For example, the constraint at the ith node could be

$$G_i = \sqrt{\frac{(\boldsymbol{T}_i)_{\max}}{\boldsymbol{T}_i}} - 1 \geq 0, \qquad i = 1, 2, \ldots, N \quad (6)$$

where $(\boldsymbol{T}_i)_{\max}$ is the prescribed temperature. A more general form of constraints could be

$$G_i = G_i(\boldsymbol{T}, \boldsymbol{\beta}) \geq 0, \qquad i = 1, 2, \ldots, N \quad (7)$$

In Eq. (5), temperatures at the nodes depend on the design parameters. In the present problem, the constraints are of the type given in Eq. (6). In the thermal sensitivity analysis, our concern is to identify the permissible variations in the design parameter values such that temperatures will not violate the constraints. When a nominal value $\boldsymbol{\beta}_{j0}$ of a parameter $\boldsymbol{\beta}_j$ is changed to $\boldsymbol{\beta}_{jc}$ such that

$$\boldsymbol{\beta}_{jc} = \boldsymbol{\beta}_{j0} + \Delta\boldsymbol{\beta}_j, \qquad j = 1, 2, \ldots \quad (8)$$

then the temperatures also change. The new temperatures can be calculated by solving the nonlinear system of Eq. (5). These new temperatures and the changed parameter values should satisfy the constraints in Eq. (7). If any constraint is not satisfied, then the calculations should be repeated with some different permissible variation in the nominal parameter value. If there are M nominal parameters whose values are likely to be changed, then the procedure has to be repeated for arbitrary variations in all of them. Only those variations that satisfy the constraints can be allowed. This procedure is extremely tedious and computationally expensive. Therefore, more sophisticated computational methods are required so that some of the quantities are calculated once for all (independent of parameter variations), and with their help the changes in the constraints due to arbitrary variations of the nominal parameter values can be obtained easily. This can be achieved if the constraint derivatives with respect to different design parameters are available. Different methods are suggested for the computation of derivatives of the constraints with respect to different parameters.

Numerical Methods

We discuss different numerical methods. In the first two methods, the objectives are 1) to calculate the temperature sensitivity functions that are used in the optimal design of a thermal control system and 2) to calculate the derivatives of the constraints that can be used to monitor the changes in the constraints for arbitrary changes in the design parameters. The third method is based on a Lagrangian approach in which the temperature sensitivity functions are not calculated and only constraint derivatives are calculated with the help of Lagrange multipliers.

It is well known that, during the life of a spacecraft, the nominal values of the design parameters change. If the design parameter β_j changes by an amount $\Delta\beta_j$ from its nominal value β_{j0} such that $\beta_{jc} = \beta_{j0} + \Delta\beta_j$, then the solution of the system of Eq. (5) will change if this new value β_{jc} is substituted into it. These new values of temperatures may or may not satisfy the constraints in Eq. (7). The variations in $\Delta\beta_j$, $j = 1, 2, \ldots, M$, are small, and it is expected that they vary within a narrow band dictated by the practical aspects of the problem. Let the change in the constraint G_i due to the parameter variation $\Delta\beta_j$ be ΔG_i. For small variations in the nominal parameter values, the following relations are approximately valid:

$$\frac{\mathrm{d}G_i}{\mathrm{d}\beta_j} \approx \frac{\Delta G_i}{\Delta\beta_j} \tag{9}$$

$$G_i|_{\beta_{jc}} \approx G_i|_{\beta_{j0}} + \left.\frac{\mathrm{d}G_i}{\mathrm{d}\beta_j}\right|_{\beta_{j0}} \Delta\beta_j \tag{10}$$

When β_{j0} is changed to β_{jc}, then the inequality given in Eq. (7) should be satisfied for the changed values. If the right-hand side (RHS) of Eq. (10) is known, then the constraint at the changed value β_{jc} can be calculated. $G_i|_{\beta_{j0}}$ is known from the problem formulation, and $(\mathrm{d}G_i/\mathrm{d}\beta_j)|_{\beta_{j0}}$ is to be calculated and can be obtained by differentiating Eq. (7) with respect to the parameter β_j:

$$\left.\frac{\mathrm{d}G_i}{\mathrm{d}\beta_j}\right|_{\beta_{j0}} = \left.\frac{\partial G_i}{\partial\beta_j}\right|_{\beta_{j0}} + \sum_{k=1}^{N} \frac{\partial G_i}{\partial \boldsymbol{T}_k}\frac{\partial \boldsymbol{T}_k}{\partial\beta_j} \tag{11}$$

On the RHS of Eq. (11), $\partial G_i/\partial\beta_j$ and $\partial G_i/\partial\boldsymbol{T}_k$ can be calculated analytically inasmuch as one constraint is considered at a time. The temperature sensitivity functions $\partial\boldsymbol{T}_k/\partial\beta_j$, $k = 1, 2, \ldots, N$, are to be calculated numerically because, generally, their analytical expression cannot be obtained. This is because the temperatures, as well as temperature derivatives at the nodes, are coupled. Once $\mathrm{d}G_i/\mathrm{d}\beta_j$ is known, then for any arbitrary variation in the nominal parameter value, the RHS of Eq. (10) is known, and it can be examined to determine whether the constraint equations are satisfied. If they are not satisfied, then a different permissible variation $\Delta\beta_j$ is taken in Eq. (10) and RHS is recalculated. This process can be repeated until $\Delta\beta_j$ satisfies all of the constraints in Eq. (7). In this process, only $\Delta\beta_j$ is to be changed in Eq. (10), and so few computations are required. If this procedure is not followed, then for every arbitrary change in the nominal parameter value, Eq. (5) must be solved, and the results checked for whether constraints are satisfied. In the discussion it has been assumed that only one parameter value changes at a time. If several parameter values are changing simultaneously, then the method described later in Eq. (17) can be used.

From the preceding discussion, it is clear that our concern is to compute the quantities $\partial\boldsymbol{T}_k/\partial\beta_j$ and $\mathrm{d}G_i/\mathrm{d}\beta_j$. As indicated earlier, these quantities can be computed in a very elementary way, which would require enormous computational time. We propose different computational methods and also present a relative study of the computational efforts involved in them.

Method 1

In this method, the emphasis is to obtain accurate temperature sensitivities. The calculation of temperature sensitivities with respect to a single parameter is described next. The temperature sensitivity functions corresponding to other parameters can be calculated in a similar way.

To calculate $\partial\boldsymbol{T}_i/\partial\beta_j$, $i = 1, 2, \ldots, N$, the following approximate relationship can be used:

$$\left.\frac{\partial\boldsymbol{T}_i}{\partial\beta_j}\right|_{\beta_{j0}} \approx \frac{\Delta\boldsymbol{T}_i}{\Delta\beta_j}, \qquad j = 1, 2, \ldots \tag{12}$$

To calculate the RHS in Eq. (12), the nonlinear equations given in Eq. (5) are to be solved to obtain the temperatures for both the nominal and the changed parameter values. From these temperatures and $\Delta\beta_j$, the $\partial\boldsymbol{T}_i/\partial\beta_j$ can be calculated. The RHS in Eq. (12) is extremely sensitive to changes in $\Delta\beta_j$. How much variation $\Delta\beta_j$ should be allowed so that the formula in Eq. (12) is fairly accurate is not clearly defined. Therefore, for a given $\Delta\beta_j$, the error should be calculated, and the formula in Eq. (12) should be accordingly corrected. The error calculation reported by Haftka and Malkus[5] will be followed here. Suppose E_i is the error associated with the computation of $\partial\boldsymbol{T}_i/\partial\beta_j$. Then

$$E_i|_{\beta_{j0}} = K_i^{(1)}\Delta\beta_j + \frac{K_i^{(2)}\zeta}{\Delta\beta_j} \tag{13}$$

where ζ is some predetermined tolerance value of successive iterations used to compute temperatures from nonlinear Eq. (5). After adding the error term, Eq. (12) can be written as

$$\frac{\partial\boldsymbol{T}_i}{\partial\beta_j} = \frac{\Delta\boldsymbol{T}_i}{\Delta\beta_j} + K_i^{(1)}\Delta\beta_j + \frac{K_i^{(2)}\zeta}{\Delta\beta_j} \tag{14}$$

In Eq. (14), $K_i^{(1)}$ and $K_i^{(2)}$ are some unknown constants. For a given i, there are three unknowns, namely, $\partial\boldsymbol{T}_i/\partial\beta_j$, $K_i^{(1)}$, and $K_i^{(2)}$. These three unknown quantities are determined with the help of three equations that can be formed by taking three different variations in β_j and calculating $\Delta\boldsymbol{T}_i/\Delta\beta_j$ corresponding to them by solving the system of equations in Eq. (5). Three values of $\Delta\beta_j$ were taken as $0.01\beta_j$, $0.05\beta_j$, and $0.1\beta_j$ ($\beta_j \neq 0$). The error term in Eq. (14) becomes minimum for a particular value of $\Delta\beta_j$, and this value can be denoted as $\Delta\beta_j|_{\mathrm{opt}}$. $\Delta\beta_j|_{\mathrm{opt}}$ can be obtained by finding the minima of the function given in Eq. (13):

$$\Delta\beta_j|_{\mathrm{opt}} = \sqrt{\frac{|K_i^{(2)}|}{|K_i^{(1)}|}\zeta} \tag{15}$$

The temperature sensitivity function for the ith node after incorporating the error can be written as

$$\frac{\partial\boldsymbol{T}_i}{\partial\beta_j} = \frac{\Delta\boldsymbol{T}_j}{\Delta\beta_j|_{\mathrm{opt}}} + K_i^{(1)}\Delta\beta_j|_{\mathrm{opt}} + \frac{K_i^{(2)}\zeta}{\Delta\beta_j|_{\mathrm{opt}}} \tag{16}$$

Different $\Delta\beta_j|_{\mathrm{opt}}$ should be generated to obtain temperature sensitivities with respect to different design parameters and different nodes. Once the temperature sensitivities are obtained, then $G_i|_{\beta_{jc}}$ and $\mathrm{d}G_i/\mathrm{d}\beta_j$ can be obtained from Eqs. (10) and (11), respectively.

In the preceding discussion, only one parameter is assumed to vary at a time from its nominal value. If several parameters are allowed to vary simultaneously, then the total variation in the constraint G_i denoted by $\mathrm{d}G_i$ due to arbitrary variations in several parameters can be obtained by calculating the total derivative of the constraint G_i

$$\begin{aligned}\mathrm{d}G_i &= \sum_{j=1}^{M}\frac{\partial G_i}{\partial\beta_j}\mathrm{d}\beta_j + \sum_{k=1}^{N}\frac{\partial G_i}{\partial\boldsymbol{T}_k}\mathrm{d}\boldsymbol{T}_k \\ &= \sum_{j=1}^{M}\frac{\partial G_i}{\partial\beta_j}\mathrm{d}\beta_j + \sum_{k=1}^{N}\frac{\partial G_i}{\partial\boldsymbol{T}_k}\sum_{j=1}^{M}\frac{\partial\boldsymbol{T}_k}{\partial\beta_j}\Delta\beta_j\end{aligned} \tag{17}$$

In Eq. (17), $\partial G_i/\partial\boldsymbol{T}_k$, $k = 1, 2, \ldots, N$, and $\partial G_i/\partial\beta_j$, $j = 1, 2, \ldots, M$, are calculated analytically, and the temperature sensitivities $\partial\boldsymbol{T}_k/\partial\beta_j$, $k = 1, 2, \ldots, N$, are to be calculated numerically as discussed earlier.

Method 2

In the second method, the temperature sensitivity functions $\partial\boldsymbol{T}_i/\partial\beta_j$, $i = 1, 2, \ldots, N$, are calculated differently, which reduces the computational time considerably.

Let the system of equations in Eq. (4) be differentiated with respect to the design parameter β_j, so that

$$\frac{\partial f_i}{\partial\beta_j} + \sum_{k=1}^{N}\frac{\partial f_i}{\partial\boldsymbol{T}_k}\frac{\partial\boldsymbol{T}_k}{\partial\beta_j} = 0, \qquad i = 1, 2, \ldots, N \tag{18}$$

Equation (18) can be written in compact form as

$$[\boldsymbol{B}] = -[\boldsymbol{A}]^{-1}[\boldsymbol{C}] \tag{19}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \dfrac{\partial f_1}{\partial \boldsymbol{T}_1}, \dfrac{\partial f_1}{\partial \boldsymbol{T}_2}, \ldots, \dfrac{\partial f_1}{\partial \boldsymbol{T}_N} \\ \dfrac{\partial f_2}{\partial \boldsymbol{T}_1}, \dfrac{\partial f_2}{\partial \boldsymbol{T}_2}, \ldots, \dfrac{\partial f_2}{\partial \boldsymbol{T}_N} \\ \cdots\cdots\cdots\cdots\cdots \\ \dfrac{\partial f_N}{\partial \boldsymbol{T}_1}, \dfrac{\partial f_N}{\partial \boldsymbol{T}_2}, \ldots, \dfrac{\partial f_N}{\partial \boldsymbol{T}_N} \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \dfrac{\partial \boldsymbol{T}_1}{\partial \boldsymbol{\beta}_j} \\ \dfrac{\partial \boldsymbol{T}_2}{\partial \boldsymbol{\beta}_j} \\ \cdots \\ \dfrac{\partial \boldsymbol{T}_N}{\partial \boldsymbol{\beta}_j} \end{bmatrix}, \qquad \boldsymbol{C} = \begin{bmatrix} \dfrac{\partial f_1}{\partial \boldsymbol{\beta}_j} \\ \dfrac{\partial f_2}{\partial \boldsymbol{\beta}_j} \\ \cdots \\ \dfrac{\partial f_N}{\partial \boldsymbol{\beta}_j} \end{bmatrix}$$

In the present problem, the components of matrix $\boldsymbol{A}$ can be obtained analytically. Matrix $\boldsymbol{B}$ is a vector of temperature sensitivity functions. To invert matrix $\boldsymbol{A}$, a numerical approach can be followed inasmuch as the inversion of matrix $\boldsymbol{A}$ by analytical means is tedious for matrices of large dimensions. The components of matrix $\boldsymbol{C}$ can be obtained analytically.

The components of matrix $\boldsymbol{A}$ involve only derivatives with respect to the temperatures of the nodes. Therefore, the analytical form of matrix $\boldsymbol{A}$ does not alter with the change in the design parameter. When a design parameter is changed, matrix $\boldsymbol{C}$ is to be computed analytically again because its components involve derivatives with respect to parameters. If the computations have been done for any one parameter, then the temperature sensitivity functions with respect to any other parameter can be computed easily because only matrix $\boldsymbol{C}$ is to be recalculated. $\boldsymbol{A}^{-1}$ can be easily calculated because its analytical form remains unchanged with the change in the parameter. An alternative method to calculate matrix $\boldsymbol{B}$ is to use the Gauss elimination method[8] in Eq. (18). It is assumed here that, for nominal parameter values, temperatures $\boldsymbol{T}_1, \boldsymbol{T}_2, \ldots, \boldsymbol{T}_N$, at all of the nodes are known. In method 1, when the temperature sensitivity functions are to be calculated with respect to a new parameter, all of the computations have to be done again. For example, the nonlinear system of Eq. (5) is to be solved numerically for the value of the changed parameter, and the error term given in Eq. (13) is to be computed again for this value of the changed parameter. However, in method 2, the computation of the derivatives of the constraints, as given in Eq. (11), remains the same as described in method 1. Method 2 is computationally more efficient than method 1.

Table 1 Comparison of different methods in terms of CPU time

Method	CPU time,[a] s	CPU time,[b] s
1	19.37	19.37
2	1.06	1.06
Lagrangian approach	3.46	0.64

[a]Corresponds to the computation of constraint derivatives of six elements given in Table 3 for two design parameters, namely, absorptance of OSR and emittance of black paint.
[b]Corresponds to the computation of constraint derivatives of a transistor plate for the two preceding design parameters.

Lagrangian Approach for Calculating Derivatives of Constraints

In this approach, the method of obtaining the derivatives of the constraints with respect to any parameter $\boldsymbol{\beta}_j$, $j = 1, 2, \ldots, N$, is entirely different from methods 1 and 2. In the first two methods, the constraint derivatives are obtained with the help of Eq. (11), but the temperature sensitivity functions are obtained differently. In the Lagrangian approach, Lagrange multipliers have been used to compute the derivatives of the constraints.

Equation (11) can be rewritten as

$$\frac{\mathrm{d}G_i}{\mathrm{d}\boldsymbol{\beta}_j} = \frac{\partial G_i}{\partial \boldsymbol{\beta}_j} + [\boldsymbol{P}][\boldsymbol{B}], \qquad i = 1, 2, \ldots, N \tag{20}$$

where $\boldsymbol{P}$ is a row vector defined as

$$[\boldsymbol{P}] = \left[\frac{\partial G_i}{\partial \boldsymbol{T}_1}, \frac{\partial G_i}{\partial \boldsymbol{T}_2}, \ldots, \frac{\partial G_i}{\partial \boldsymbol{T}_N}\right] \tag{21}$$

and matrix $[\boldsymbol{B}]$ is defined in Eq. (19). The components of vector $\boldsymbol{P}$ can be obtained analytically. On substituting $-[\boldsymbol{A}]^{-1}[\boldsymbol{C}]$ in place of $[\boldsymbol{B}]$, Eq. (20) becomes

$$\frac{\mathrm{d}G_i}{\mathrm{d}\boldsymbol{\beta}_j} = \frac{\partial G_i}{\partial \boldsymbol{\beta}_j} - [\boldsymbol{P}][\boldsymbol{A}]^{-1}[\boldsymbol{C}], \qquad i = 1, 2, \ldots, N \tag{22}$$

In Eq. (22), the analytical expressions of the components of matrices $[\boldsymbol{P}]$ and $[\boldsymbol{A}]$ are not changing with changes of the parameters, as the differentiations are not with respect to design parameters, but matrix $[\boldsymbol{C}]$ is changing. If the number of constraints is lower than the number of parameters, i.e., $N < M$, then it is possible to obtain the constraint derivatives with respect to all M parameters by solving the system of equations only N times rather than M times. Multiplication of $[\boldsymbol{P}][\boldsymbol{A}]^{-1}[\boldsymbol{C}]$ in Eq. (22) can be simplified with the help of Lagrange multipliers. Consider an auxiliary row vector

$$\varphi = [\varphi_1, \varphi_2, \ldots, \varphi_N]$$

where $\varphi_1, \varphi_2, \ldots, \varphi_N$ are the Lagrange multipliers. The vector φ is obtained by solving the following system of equations:

$$[\varphi][\boldsymbol{A}] = [\boldsymbol{P}] \tag{23}$$

In Eq. (23), matrices $[\boldsymbol{A}]$ and $[\boldsymbol{P}]$ are given by Eqs. (19) and (21), respectively. In terms of Lagrange multipliers, the derivative of any constraint with respect to any parameter can be obtained from the following equation:

$$\frac{\mathrm{d}G_i}{\mathrm{d}\boldsymbol{\beta}_j} = -[\varphi][\boldsymbol{C}], \qquad i = 1, 2, \ldots, N \tag{24}$$

In the present problem $\partial G_i/\partial \boldsymbol{\beta}_j = 0$. Note that, in Eq. (24), Lagrange multipliers are to be calculated only once for every constraint under study, and the same values of the Lagrange multipliers can be used to obtain the derivative of the constraint with respect to the different parameters. However, for different constraints, different sets of Lagrange multipliers are needed. It has become evident that the Lagrangian approach is computationally economical in a situation where the number of constraints is lower than the number of parameters, i.e., when $N < M$. If $N > M$, then method 2 seems to be computationally more economical than the Lagrangian approach. When $N = M$, there does not seem to be much difference in the computational efforts between method 2 and the Lagrangian approach.

To give some idea about the relative computational effort, a comparison of computational time needed for the different methods is given in Table 1. As discussed earlier, in method 1, the nonlinear equations are to be solved to obtain temperature sensitivities, and errors associated with them are to be calculated. This increases

Table 2 Nominal values of design parameters

Design parameter	Absorptance	Emittance	Component	Internal heat dissipation, W
OSR	0.08	0.78	Control CKT PCB	4.41
Black paint	0.90	0.90	Driver CKT PCB	4.00
Low-emittance tape	0.15	0.05	Diode plate	18.90

the computational time. In method 2, temperature sensitivities are obtained by solving a linear system of equations. When the number of elements is greater than the number of design parameters, i.e., $N > M$, then it is clear from the second column of Table 1 that method 2 is more economical than the Lagrangian approach. If $N < M$, which is the case in third column of Table 1, then the Lagrangian approach is more economical. The computations were done on an IBM RS6000.

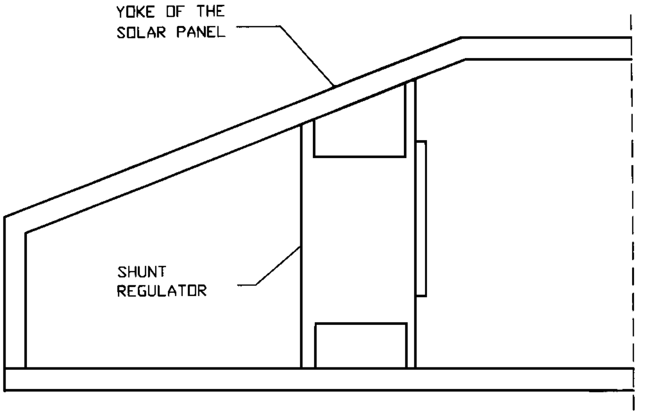

Fig. 1 Location of shunt regulator package in spacecraft.

Description of the Shunt Regulator

The shunt regulator of the spacecraft is mounted on the yoke of the solar panel, as shown in Fig. 1. It comprises four major components, namely, the transistor plate, control circuit (CKT) printed circuit board (PCB), driver CKT PCB, and diode plate. Heat dissipation on these components is supplied by the solar panel. The shunt regulator is subjected to external solar loads varying with the seasons. The thermal control system used for the nominal design is shown in Fig. 2. The OSR has been provided on the sun-facing surface of the shunt regulator. The four side surfaces are covered with low-emittance tape. The antisun-facing side of the shunt regulator is treated partially with black paint, and the rest of the area is covered with low-emittance tape.

Discussion of Temperatures and Their Sensitivities

The components of the shunt regulator are to be maintained within the temperature range of −45–65°C so that the temperatures satisfy the requirements of internal heat dissipation and the external solar load. Internal heat dissipations for the components are given in Table 2. Solar load has been considered with the solar intensity as in the summer solstice. The solar absorptivities considered in the computations correspond to the beginning of the life of the spacecraft. In Table 3, the temperatures of the components of the shunt regulator corresponding to nominal design parameter values are given together with their temperature margins, which are the differences between the upper prescribed design temperature and the nominal

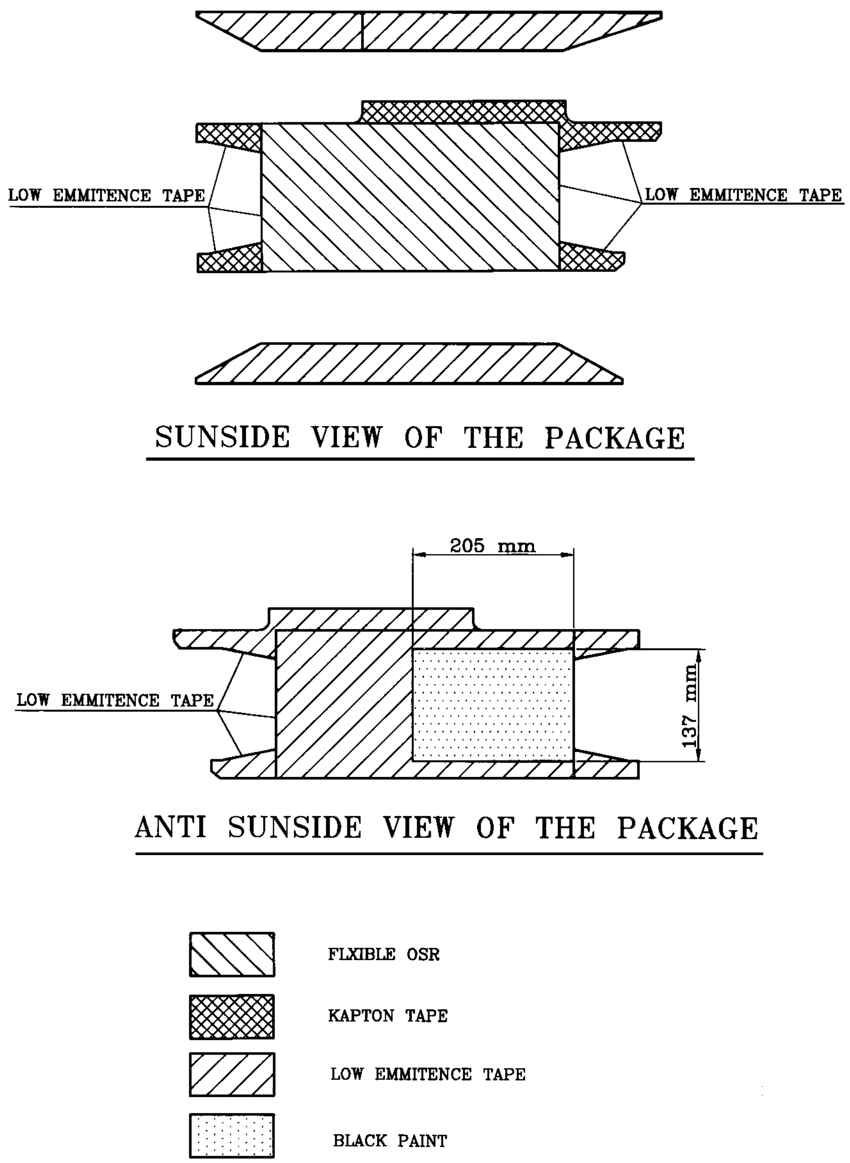

Fig. 2 Sunside and antisunside views of shunt regulator.

design temperatures. The nominal temperatures are obtained by solving the nonlinear system of equations in Eq. (5). It can be seen from Table 3 that the temperature margins for some of the components, such as the control CKT PCB, driver CKT PCB, and diode plate, are less than 15°C. It is desirable to reduce the temperatures of the components (or, in other words, to increase the temperature margins for these components) so that a higher tolerance of temperatures can be assigned to the variation of the design parameters. To reduce the temperatures, it becomes necessary to study the temperature sensitivities of the components with respect to the design parameters.

In Table 4, the temperature sensitivities of the components with respect to absorptance of OSR, emittance of OSR, emittance of black paint, emittance of low-emittance tape, and internal heat dissipation are presented. Methods 1 and 2 have been used to calculate temperature sensitivities. The sensitivities shown in parentheses correspond to those obtained from method 2, whereas those without parentheses correspond to method 1. Sensitivities having negative coefficients indicate that increases in the respective parameters help in reducing temperatures, and sensitivities having positive coefficients indicate that increases in the respective design parameters increase the temperature of the components.

To select the design parameter that has the maximum effect in reducing the temperatures, the semirelative sensitivity functions[2,9] should be calculated. Sensitivities corresponding to different design parameters will have different units. Therefore, sensitivities cannot be compared. However, they can be compared with the help of semirelative sensitivity functions in which the units remain the same for all of the design parameters. Semirelative sensitivities corresponding to all of the design parameters considered are shown in Table 5. In this case, they are calculated by multiplying the sensitivities by the nominal values of the design parameters. Semirelative sensitivity functions reveal that the increase in emittance of OSR helps in decreasing the nominal temperature of the components to the maximum extent. Because there is no scope of increasing the emittance of OSR because of design constraints, the other semirelative sensitivities should be studied. The next dominant parameter for reducing the temperature is found to be internal heat dissipation. The internal heat dissipation in the shunt regulator is generally dictated by the solar panel and, therefore, reducing the temperatures by decreasing the value of internal heat dissipation is not possible.

The next dominant parameter, as seen from Table 5, is the absorptance of OSR. The reduction of absorptance of the OSR is found to help reduce the temperatures as solar heat input to the shunt regulator is reduced. The solar heat input can be reduced with the help of a change in the design in the form of a thermal shield. If this thermal shield is provided in front of the sun-facing side of the shunt regulator with its sun-facing surface covered with the OSR and the other surface painted black, then the solar heat input is certainly

Table 3 Nominal temperatures and their margins before incorporating suggested design

Component of shunt regulator	Temperature,[a] °C	Temperature margin,[b] °C
Transistor plate	28.5	36.5
Control CKT PCB	55.0	10.0
Driver CKT PCB	50.2	14.8
Diode plate	51.9	13.1
Top cover	22.9	42.1
Bottom cover	28.2	36.8

[a]Correspond to nominal design parameter values.
[b]Difference between upper prescribed design temperature and the nominal design temperature.

Table 6 Temperatures and their margins after incorporating suggested design changes

Component of shunt regulator	Temperature,[a] °C	Temperature margin,[b] °C
Transistor plate	14.4	50.6
Control CKT PCB	44.2	20.8
Driver CKT PCB	41.7	23.3
Diode plate	36.0	29.0
Top cover	15.9	49.1
Bottom cover	13.6	51.4

[a]Correspond to suggested design changes.
[b]Difference between upper prescribed design temperature and the temperature of the suggested design.

Table 4 Temperature sensitivities obtained from methods 1 and 2

Component	Absorptance of OSR, K	Emittance of OSR, K	Emittance of black paint, K	Emittance of low-emittance tape, K	Internal heat dissipation, K/W
Transistor plate	307.0	−58.3	−21.9	−87.9	1.93
	(308.4)[a]	(−59.1)	(−22.2)	(−88.2)	(1.94)
Control CKT PCB	273.2	−53.8	−13.2	−85.4	1.54
	(281.8)	(−54.0)	(−12.7)	(−72.9)	(1.58)
Driver CKT PCB	270.0	−51.9	−14.6	−68.9	1.41
	(274.0)	(−52.5)	(−14.3)	(−64.6)	(1.42)
Diode plate	322.4	−63.1	−18.1	−106.3	3.04
	(330.1)	(−63.2)	(−17.7)	(−96.6)	(3.08)
Top cover	383.1	−73.8	−13.1	−74.0	1.55
	(387.8)	(−74.3)	(−13.3)	(−72.1)	(1.58)
Bottom cover	320.9	−58.0	−21.0	−75.3	2.03
	(313.6)	(−60.1)	(−22.0)	(−90.8)	(2.00)

[a]Values in parentheses refer to method 2.

Table 5 Semirelative sensitivity functions for various design parameters

Component	Absorptance of OSR, K	Emittance of OSR, K	Emittance of black paint, K	Emittance of low-emittance tape, K	Internal heat dissipation, K
Transistor plate	24.56	−45.47	−19.71	−4.39	36.48
	(24.67)	(−46.09)	(−19.98)	(−4.41)	(36.66)
Control CKT PCB	21.86	−41.96	−11.88	−4.27	29.11
	(22.54)	(−42.12)	(−11.43)	(−3.64)	(29.86)
Driver CKT PCB	21.60	−40.48	−13.14	−3.45	26.65
	(21.92)	(−40.95)	(−12.87)	(−3.23)	(26.83)
Diode plate	25.79	−49.22	−16.29	−5.32	57.46
	(26.40)	(−49.29)	(−15.93)	(−4.83)	(58.21)
Top cover	30.65	−57.56	−11.79	−3.70	29.29
	(31.02)	(−57.95)	(−11.97)	(−3.60)	(29.86)
Bottom cover	25.67	−45.24	−18.90	−3.76	38.37
	(25.08)	(−46.87)	(−19.80)	(−4.54)	(37.80)

Table 7 Comparison of constraint derivatives obtained by different methods

Component	Absorptance of OSR, K	Emittance of OSR, K	Emittance of black paint, K	Emittance of low-emittance tape, K	Internal heat dissipation, K/W
Transistor plate	−0.5347[a]	0.1015	0.03815	0.15311	−0.00336
	−0.5372[b]	0.1029	0.03867	0.15363	−0.00338
	−0.5565[c]	0.1066	0.04037	0.15524	−0.00335
Control CKT PCB	−0.4191	0.0826	0.02026	0.13110	−0.00236
	−0.4326	0.0829	0.01950	0.11190	−0.00243
	−0.5575	0.1068	0.02480	0.13852	−0.00294
Driver CKT PCB	−0.4238	0.0815	0.02291	0.10813	−0.00221
	−0.4300	0.0824	0.02244	0.10139	−0.00223
	−0.5528	0.1059	0.02889	0.12683	−0.00274
Diode plate	−0.5021	0.0982	0.02819	0.16550	−0.00478
	−0.5140	0.0984	0.02756	0.15043	−0.00480
	−0.5377	0.1030	0.02878	0.15305	−0.00480
Top cover	−0.6863	0.1322	0.02347	0.13257	−0.00278
	−0.6947	0.1331	0.02383	0.12917	−0.00283
	−0.6948	0.1330	0.02380	0.12607	−0.00270
Bottom cover	−0.5598	0.1012	0.03663	0.13136	−0.00354
	−0.5471	0.1048	0.03838	0.15840	−0.00349
	−0.5621	0.1076	0.03952	0.15882	−0.00344

[a]Constraint derivatives in first row for every component correspond to method 1.
[b]Constraint derivatives in second row for every component correspond to method 2.
[c]Constraint derivatives in third row for every component correspond to the Lagrangian approach.

reduced, but the effective radiating capacity of the shunt regulator to space is also reduced. Therefore, the thermal shield cannot be provided unless the radiating capacity of the shunt regulator is improved. To improve the radiating capacity of the shunt regulator, the black-painted area on the antisun side can be increased. The additional black-painted area on the antisun side can be obtained only by numerical experimentations and observing the temperatures of the components at extreme heat load conditions. An additional black-painted area required in the present study is about 109.5% of the area of black paint in the nominal design.

In Table 6, the temperatures have been presented that pertain to the suggested design of introducing a thermal shield and increasing the black-painted area. It is clear from Table 6 that the temperatures corresponding to the suggested design have been considerably reduced as compared with the temperatures given in Table 3 for the nominal design. This decrease in temperature values increases the temperature margins, thus resulting in better performance of the system. The increase in black-painted area, which is not a design parameter, can be determined only by trials such that maximum temperature margins are obtained. In Table 7, a comparison of constraint derivatives obtained by methods 1 and 2 and the Lagrangian approach has been presented. For the new suggested design, the Lagrangian approach was found to be convenient because the changes in the design parameters can be easily incorporated.

Conclusions

From the preceding discussion, the following conclusions can be drawn about the relative merits of the numerical methods employed and the thermal control of the shunt regulator.

1) It is clear from Table 4 that the temperature sensitivities calculated with methods 1 and 2 are comparable. However, the computational effort in method 1 is greater than in method 2. Method 1 requires the solution of nonlinear equations for which generally iterative methods are employed, which increases the computational time. Further, additional computations are required in method 1 to calculate the errors associated with the temperature sensitivities. In method 2, the equations to be handled are linear, whose solution requires much less computational effort compared with method 1, and so method 2 will always be more economical than method 1.

2) The computation of temperature sensitivities is needed in methods 1 and 2. The choice between method 2 and the Lagrangian approach with respect to the computational effort depends on the ratio of the number of nodes to the number of design parameters. If the number of nodes N is greater than the number of design parameters M, then method 2 involves less computational effort compared with the Lagrangian approach. This is because in method 2, Eq. (19) is to be solved M times, whereas in the Lagrangian approach, N Lagrange multipliers are to be calculated for each node. Therefore, if the number of nodes is smaller than the number of design parameters, the Lagrangian approach will be preferred. If $N = M$, then method 2 and the Lagrangian approach involve roughly the same computational effort.

3) The temperature sensitivities in Table 4 indicate that the temperature of the shunt regulator can be reduced by reducing the absorptance of the OSR. However, reduction in the absorptance of the OSR seems difficult due to the nonavailability of an alternate material that has less absorptance and the same emittance as that of the OSR. The desired objective can be achieved with the help of some design modifications. Two design modifications are suggested: 1) a thermal shield in front of the sun-facing surface of the shunt regulator and 2) an increase in the radiating area by increasing the black-painted area on the antisun side of the shunt regulator. It is clear from Table 6 that the suggested design reduces the nominal design temperatures and, thus, improves the performance of the system.

Acknowledgments

S. Suresha is thankful to H. Narayanamurthy, Group Director, Thermal Systems Group (TSG), Indian Space Research Organisation (ISRO) Satellite Centre, Bangalore, India, and R. A. Katti, Head, Thermal Analysis Section, TSG, ISRO Satellite Centre, Bangalore, India, for their constant encouragement.

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Associate Editor